PERFORMANCE OF BASIC VECTOR DIRECTIONAL FILTERS ACCORDING TO USED ANGLE DISTANCE

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Abstract

Color images are typical examples of vector-valued signals. For that reason, vector processing represents an optimal approach. Although widely used vector filter is a vector median based on a reduced ordering, the directional processing with utilising of the angle between input vectors can be used, too. By this way can be achieved well estimates, since vector directional filters preserve color chromaticity, whereas vector median filters may not satisfy this requirement. So, this paper is focused on the performance of basic vector directional filter in dependence on the various angle distances.

Keywords

Color images, vector filters, directional filters, impulse noise, Gaussian noise

1. Introduction

At present, processing of color images is related with a number of applications such as television, communication technologies, computers, robot visions, medical imaging, etc. Thus, the considerable attention is focused on the preserving of color information [8] on that the human visual system is most sensitive. In case of noise corruption [3], [5], [7], [11], [16], this problem concerns filter algorithms.

Recently developed class of vector directional filters [14,15] is very important, since to produce an estimate, the directional information based on angle distance of input vectors is utilised. By this way, the directional processing represents optimal approach according to color images that are vector-valued signals, and thus, they may be processed according to angle and magnitude distance of input vectors. Widely used vector median-type filters based on reduced ordering utilise magnitude processing, only, what must not preserve color chromaticity. For that reason, in some situations it is most appropriate to use vector filters based on directional processing.

This paper is organised as follows. In Section II, fundamentals of directional processing including various types of angle distances are presented. The recently developed directional vector filters are formulated in Section III, where is defined basic vector directional filter, general vector directional filter and spherical median filter. Section IV includes a number of experimental results. Especially, this Section is focused on the performance of basic vector directional filters. Finally, objectives are summarised in Section V.

2. Directional processing

In the vector space, image points of color images, i.e. vector-valued or multivariate samples, are determined by their magnitude and angle. Concerning the magnitude only, it is the case of magnitude processing. The typical example of vector filters based on magnitude processing are vector median-type filters such as vector medians, extended vector medians and weighted vector median filters. Another case of filters based on magnitude processing is given by component-wise filters that utilize the decomposition of color channels and thus, these filters are applied on the each color channel separately.

Vector directional filters (VDF) [3], [5], [7], [10], [11], [14], [15] are based on directional information, and thus, vector magnitudes are taken in the account of vector directions, i.e. angles, only. Importance of VDF resides in processing concept, where vector directions indicate color chromaticity. Simply, VDF perform optimal filtering operation in the sense of sample direction preservation.

Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$ characterize input set and N is a window size. Each sample $\mathbf{x}_i$ (for $i = 1, 2, \ldots, N$) represents m-dimensional vector. In the case of color images, $m = 3$, since image points are represented by tristimulus values. In general, the difference between two m-dimensional vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ can be expressed not only by the comparing of vector magnitude or separately elements (the case of absolute and Euclidean distances), however, the angle $A(\mathbf{x}_i, \mathbf{x}_j)$ between two m-dimensional vectors $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{im},)$ and $\mathbf{x}_j = (x_{j1}, x_{j2}, \ldots, x_{jm},)$ defined as

$$\mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) = \cos^{-1}\left(\frac{\mathbf{x}_i . \mathbf{x}_j^T}{|\mathbf{x}_i| . |\mathbf{x}_j|}\right) \tag{1}$$

$$= \cos^{-1}\left(\frac{x_{i1}x_{j1} + x_{i2}x_{j2} + \ldots + x_{im}x_{jm}}{\sqrt{x_{i1}^2 + x_{i2}^2 + \ldots + x_{im}^2}\sqrt{x_{j1}^2 + x_{j2}^2 + \ldots + x_{jm}^2}}\right) \tag{2}$$

is used, too. In general case $0 \leq A(\mathbf{x}_i, \mathbf{x}_j) \leq \pi$ is valid, whereas in case of color image [14], [15] $0 \leq A(\mathbf{x}_i, \mathbf{x}_j) \leq \pi/2$.

- *Absolute angle distance*

If $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$ define input set, then absolute angle distance α_i that corresponds with vector sample $\mathbf{x}_i$ can be expressed as

$$\alpha_i = \sum_{j=1}^{N} A(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1,2,...,N \tag{3}$$

where $A(\mathbf{x}_i, \mathbf{x}_j)$ denotes the angle between vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ (1). Another expressions of angle distance can be achieved introducing mean and square operators, what is performed as follows [11]:

- *Mean absolute angle distance*

$$\alpha_i = \frac{1}{N}\sum_{j=1}^{N} A(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1,2,...,N \tag{4}$$

- *Square angle distance*

$$\alpha_i = \sum_{j=1}^{N} A^2(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1,2,...,N \tag{5}$$

- *Mean square angle distance*

$$\alpha_i = \frac{1}{N}\sum_{j=1}^{N} A^2(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1,2,...,N \tag{6}$$

It is evident that the performance of vector filters based on directional information depends on the type of used angle distance. These merits can be utilized in various filtering algorithms based on directional information. According this, a class of vector directional filters should have to offer the same wide possibilities as in the case of filters based on magnitude processing. For that reason, it is interested to list some directional techniques. So, in next Section is presented the basic description of most frequently used vector directional filters.

3. Vector directional filters

To define an estimate of vector directional filters, the ordering process of multivariate samples $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$ according to associated angle distances $\alpha_1, \alpha_2, \ldots, \alpha_N$ must be described. Let α_i (for $i = 1, 2, \ldots, N$) characterizes angle distances associated with input vector-valued sample $\mathbf{x}_i$ according to definitions from (3) to (6). Then the ordered set of angle distances $\alpha_1, \alpha_2, \ldots, \alpha_N$ can be expressed as [7], [14], [17]

$$\alpha_{(1)} \leq \alpha_{(2)} \leq \ldots \leq \alpha_{(r)} \leq \ldots \leq \alpha_{(N)} \tag{7}$$

To achieve an ordered input set, the same ordering is required being applied to corresponding input samples $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$, that can be expressed as

$$\mathbf{x}^{(1)} \leq \mathbf{x}^{(2)} \leq \ldots \leq \mathbf{x}^{(r)} \leq \ldots \leq \mathbf{x}^{(N)} \tag{8}$$

After this and according to principle of used vector directional filters, the choice of estimate can be performed as a sample or set of samples from ordered set $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(N)}$.

3.1 Basic vector directional filter

If the first sample from ordered set (8) is chosen to represent the filter output, this filtering operation is so called as basic vector directional filter (BVDF). Mathematically, mentioned operation can be can expressed as [11], [14], [15]

$$\mathbf{y}_{BVDF} = \mathbf{x}^{(1)} \tag{9}$$

BVDF outputs the sample from input set that minimizes the sum of angles with other vectors. This operation, the color distortion is reduced, since BVDF does not produce new and unnatural samples that usually result in color artefacts.

3.2 General vector directional filter

If a filter output can be expressed as the set of first r terms of (8) what is given by

$$\mathbf{y}_{GVDF} = \left\{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, ..., \mathbf{x}^{(r)}\right\} \tag{10}$$

it is called general vector directional filter (GVDF) [7], [14]. Output set $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(r)}$ of GVDF has approximately the equal direction in a vector space and thus, it can be used as an input for additional filter [10], [11], where $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(r)}$ are processed according to their magnitude.

Usually, α-trimmed average filter, multistage median filter and some morphological filters are used. It follows that GVDF differentiate processing of vector-valued samples on directional processing and magnitude processing.

From theoretical properties [15], both BVDF and GVDF filters are invariant to under scaling and rotation, however, not invariant to bias.

3.3 Spherical median filter

Spherical medians (SMF) [14], [15] belong to another subclass of VDF. Filtering algorithm of is based on the minimizing of following expression

$$\alpha = \sum_{j=1}^{N} A(\mathbf{x}_S, \mathbf{x}_j) \tag{11}$$

where A(., .) characterizes angle between two vectors.

However, unlike BVDF and GVDF filters, SMF ones produce new sample as filter output, since output of SMF is represented by random vector $\mathbf{x}_S = \{x_{S1}, x_{S2}, \ldots, x_{Sm},\}$ that minimizes (11), where $\mathbf{x}_j$ (for $j = 1, 2, \ldots, N$) represents input set. Thus, the estimate of SMF is usually accompanied with some error that results in color artefact with the simultaneous angle distortion of output vector.

Fig. 1 (a) Original image Lena, (b) Original image Mandrill, (c) 2% impulse noise (I2), (d) I2 filtered by vector median, (e) I2 filtered by BVDF with absolute angle, (f) Gaussian noise $\sigma = 16$ (G16), (g) G16 filtered by component-wise median, (h) G16 filtered by BVDF with absolute angle.

4. Experimental results

Although three classes of vector directional filters were described in previous Section, the aim of this paper is focused on the performance of BVDF (9) only. Original images Lena and Mandrill are show in Fig. 1a-b. To simulate a noise corruption [3], [6], [7], [16], two noise models were used, namely impulse noise with uniform distribution and Gaussian noise, both were separately generated for each color channel, what results in noncorrelated noise type. Achieved results and differences from original images were evaluated by well-known mean absolute error (MAE), mean square error (MSE) and color difference (CD) [3], [7], [12]. In case of MAE and MSE, these criteria are taken as mean over 3 color channels.

Concerning used angle distance, the best results were achieved under the absolute angle (3). $BVDF_1$ characterises basic vector directional filter with absolute distance (3), $BVDF_2$ performs BVDF operations based related with use of mean absolute distance (4), $BVDF_3$ and $BVDF_4$ are used to describe BVDF filter with square distance (5) and mean square angle distance (6), separately.

Achieved results are summarised in Tab. 1 to 4. Next, subjective visualisation of filter performance is provided on Fig. 1c to h. In term of color difference (CD) it can be seen that for lowly corrupted images BVDF filters with absolute distance (3) produce better estimates in comparison with vector median filter (VMF) [1,2,4] and component-wise median (MF) [4]. The fact, that in comparison with standard absolute and square angle distances the mean distances do not bring the result improvement, is very important.

On the other hand, in case of MAE and MSE, vector BVDF filters do not achieve better noise suppression than that of vector median. It's caused by BVDF filtering algorithm preserving optimal color chromaticity, how-

Noise	2%			10%		
Method	*MAE*	*MSE*	*CD*	*MAE*	*MSE*	*CD*
identity	1.519	173.0	7.060	7.312	832.0	32.717
MF	3.239	45.9	15.121	3.703	56.8	17.777
VMF	3.300	47.8	14.029	3.687	56.5	15.396
$BVDF_1$	3.694	55.1	14.051	4.099	67.6	15.343
$BVDF_2$	3.694	55.1	14.051	4.099	67.6	15.343
$BVDF_3$	4.847	89.5	17.580	6.691	170.0	23.117
$BVDF_4$	4.847	89.5	17.580	6.691	170.0	23.117

Tab. 1 The impulse noise suppression on image Lena.

Noise	2%			10%		
Method	*MAE*	*MSE*	*CD*	*MAE*	*MSE*	*CD*
identity	1.509	171.7	6.918	7.221	811.4	33.431
MF	10.135	267.9	30.537	10.647	288.4	33.629
VMF	10.336	278.5	29.241	10.818	297.2	30.634
$BVDF_1$	13.320	470.7	27.913	13.688	499.0	29.099
$BVDF_2$	13.320	470.7	27.913	13.688	499.0	29.099
$BVDF_3$	14.005	495.0	31.295	15.431	594.4	37.085
$BVDF_4$	14.005	495.0	31.295	15.431	594.4	37.085

Tab. 2 The impulse noise suppression on image Mandrill.

Noise	$\sigma = 16$			$\sigma = 32$		
Method	*MAE*	*MSE*	*CD*	*MAE*	*MSE*	*CD*
identity	12.340	246.2	80.851	24.193	928.3	159.34
MF	7.155	96.8	38.971	11.487	221.2	67.769
VMF	8.610	132.7	49.038	14.502	343.9	88.577
$BVDF_1$	11.350	245.7	43.934	19.131	635.5	76.557
$BVDF_2$	11.350	245.7	43.934	19.131	635.5	76.557
$BVDF_3$	11.310	244.5	43.305	19.016	627.7	74.740
$BVDF_4$	11.310	244.5	43.305	19.016	627.7	74.740

Tab. 3 The Gaussian noise suppression on image Lena.

Noise	$\sigma = 16$			$\sigma = 32$		
Method	*MAE*	*MSE*	*CD*	*MAE*	*MSE*	*CD*
identity	12.108	235.8	65.436	24.294	939.8	134.57
MF	12.543	309.5	42.949	15.924	440.9	63.409
VMF	13.865	363.7	50.105	18.678	590.4	79.621
$BVDF_1$	18.500	679.2	43.716	24.454	1060.5	68.669
$BVDF_2$	18.500	679.2	43.716	24.454	1060.5	68.669
$BVDF_3$	18.442	669.1	43.999	24.435	1057.7	67.459
$BVDF_4$	18.442	669.1	43.999	24.435	1057.7	67.459

Tab. 4 The Gaussian noise suppression on image Mandrill.

ever in general, BVDF filters cannot achieve the noise attenuation comparable with vector median filters. Mentioned property is independent on the degree of the noise corruption, since BVDF filters achieve worse results in case of lowly corrupted images (impulse noise with impulse probability $p = 0.02$ and the Gaussian noise with $\sigma = 16$), too.

5. Conclusion

The aim of this paper was oriented on the use of basic vector directional filters for filtering of noisy color images, i.e. vector-valued signals. The performance of proposed basic vector directional filters was tested according to four used angle distances and two noise types with wide range of noise corruption. Achieved results signify the seemliness of directional or angle processing in the case of both, impulse noise and Gaussian noise on the ground of preserving of color chromaticity. It can be summarised that basic vector directional filters produce results, where natural color tones are preserved. For that reason, BVDF are well evaluated at subjective tests, since human eye is sensitive on color artefact. On the other hand, insufficient perfor mance of the noise corruption can restrict the use of directional information to estimate of origin vector sample.

Worse signal details preservation and noise suppression obtained by BVDF are due to, since magnitude information carried by the vector is not taken into account when computing a filter output. Important is the fact, that basic vector directional filters are characterized by a robustness, what means the same performance at statistical various areas.

To improve the noise attenuation and thus, to enable a widely use in smoothing applications, two approaches are offering. The first is based on adaptive determination of a filter output, where could be used neural networks [7] or fuzzy decisions [11]. The second approach leads to weighted structures [18]. By this way, it is possible to achieve the same impulse noise attenuation as in the case of weighted componentwise filters and weighted vector median filters.

Both above approaches are fully applicable in directional processing and they can disclose potential embedded in directional information of a vector space.

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